

TRAJECTORIES OF SOLAR SATELLITE FOR RADIO SOUNDING  
OF CIRCUMSOLAR SPACE

G. A. Mersov

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16. Abstract  A heliocentric orbit, symmetrical to the orbit of the Earth with reference to the center of the Sun, and called a contra-Earth orbit is investigated. While a space sound moves along a contra-Earth orbit, the Sun, the Earth and the sound gradually form a straight line which can be used to carry out experiments relating to the radio sounding or circumsolar space. Methods of putting the sound into a contra-Earth orbit are suggested and the magnitudes of velocity of the inserting maneuvers are defined. The perturbed motion of the sound is investigated and the demands for precision in carrying out the insertion maneuvers in limiting circular perturbation are defined.			
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# TRAJECTORIES OF SOLAR SATELLITE FOR RADIO SOUNDING OF CIRCUMSOLAR SPACE

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## Introduction

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Observing the turn of a space probe behind the Sun or its passage near the solar disc presents possibilities for carrying out experiments based on radio sounding of circumsolar space. Heliocentric orbits, guaranteeing periodic turns or junctions of the probe have attracted the attention of researchers more than once [1-2]. The most interesting orbits for an Earth observer gradually approach the disc of the Sun in their movement. Such orbits can be achieved by means of perturbation of an orbit symmetrical to the orbit of the Earth with reference to the Sun (Figure 1). We shall refer to this orbit below as a contra-Earth orbit. It is obvious that all the elements of a contra-Earth orbit, except the amplitude of perihelion, coincide with elements of the orbit of the Earth, and the perihelion amplitude differs from the perihelion amplitude of the orbit of the Earth by  $180^\circ$ . If a probe is inserted into a contra-Earth orbit at the exact spot which corresponds to the Earth at the moment of insertion, further motion brings the Earth, the Sun, and the probe into a straight line with a precision equal to the evolution of the orbits of the Earth and the probe. When this motion is perturbed, the probe will move for an earthly observer into the celestial sphere with reference to the disc of the Sun. Eliminating the secular perturbations makes it possible to limit the relative movement of the probe to a defined vicinity of the solar disc.

In this work methods of inserting a probe into a contra-Earth orbit are proposed, the perturbation motion of the probe is investigated, and the requirements for precision in the insertion with a limitation on the secular perturbations are defined. In this respect we assume that:

1) the motion of the Earth and of the probe occur only through the action of the gravitational field of the Sun;

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\*Numbers in the margin indicate pagination in the foreign text.

2) the radius of the sphere of activity of the Earth is negligibly small in comparison with the distance from the Earth to the Sun;

3), all insertion maneuvers are carried out instantaneously.

### 1. Inserting a Probe into a Contra-Earth Orbit

We shall examine two methods of inserting a probe, using in each method two intermediate orbits A and B with the lines of their apsides coinciding with the line of apsides of the orbit of the Earth. In the first method, Figure 2, the sound shifts from the orbit of the Earth to orbit A in the perihelion orbit of the Earth, and from orbit B to an intermediate orbit in the aphelion contra-Earth orbit. In the second method, Figure 3, the probe shifts to orbit A in the aphelion of the orbit of the Earth, and from orbit B to an intermediate orbit in its perihelion. In both cases the moment of insertion of the probe into the intermediate orbit at point Z, the Earth should be at point 4, symmetrical to point 3 with reference to the Sun. Subsequently the duration of motion of the probe from the moment of launching from the orbit of the Earth at point 1 to the moment of insertion in the intermediate orbit at point 3 should be equal to the length of motion of the Earth from point 1 to point 4  $\tau$ :

$$\tau_a + \tau_b = \tau,$$

where  $\tau_a$  and  $\tau_b$  are the lengths of time the probe is in orbit A and in orbit B respectively. This equation can be written in the form

$$k(r_1 + r_2)^{\frac{3}{2}} + l(r_2 + r_3)^{\frac{3}{2}} = m(r_1 + r_3)^{\frac{3}{2}}, \quad (1)$$

where k and  $\tau$  are the number of half-turns completed by the probe in orbits A and B respectively, and m is the number of half-turns completed by the Earth in the process of moving from point 1 to point 4. In conformity with the methods of insertion observed k, l and m are arbitrarily odd numbers. Equation (1) for the given k, l and m define the radius  $r_2$  of point 2, at which the probe shifts from orbit A to orbit B. The magnitude m is defined by the total duration of the flight of the probe to insertion  $\tau$ , equal to  $m/2$  years. With the given magnitude m, the magnitudes k and l are limited from above in conformity with the following inequality

$$k r_1^{\frac{3}{2}} + l r_3^{\frac{3}{2}} \leq m (r_1 + r_3)^{\frac{3}{2}} \quad (2)$$

Condition (2) appears as a consequence of the presence of the intermediate orbits with least period, achieved at  $\Gamma_2$  equal to zero. In detecting all magnitudes  $k$  and  $l$  satisfying condition (2) with the given magnitude  $m$ , we can determine with the help of (1) all possible intermediate orbits providing insertion into contra-Earth orbit after  $m/2$  years. From these findings we determine those which require the least expenditure for insertion, as an optimal quality. In terms of characteristic expenditures for insertion we shall use the magnitude of the sum of the velocities of all maneuvers necessary for inserting the probe into the contra-Earth orbit.

In the general case the methods examined assume the execution of 3 maneuvers. The first maneuver takes care of inserting the probe into intermediate orbit A. Watching the probe from the circular orbit of an Earth satellite with a radius  $\Gamma_g$ , the velocity of the first maneuver will be equal to

$$V_g = \sqrt{V_1^2 + \frac{2\mu_g}{\Gamma_g}} - \sqrt{\frac{\mu_g}{\Gamma_g}},$$

where  $\mu_g$  is the gravitational parameter of the Earth, and  $V_1$  the velocity of the probe with reference to the Earth on the sphere of activity of the Earth, necessary for the probe to enter orbit A:

$$V_1 = \sqrt{2\mu_s r_s / (r_1 + r_s) r_1} - \sqrt{2\mu_s r_s / (r_1 + r_s) r_s},$$

where  $\mu_s$  is the gravitational parameter of the Sun. The second and third maneuvers respectively guarantee passage of the probe from orbit A to orbit B and from orbit B to the contra-Earth orbit. As a result of carrying out these maneuvers, the velocity of the probe with reference to the Sun should be changed to magnitudes  $V_2$  and  $V_3$ :

$$\begin{aligned} V_2 &= \sqrt{2\mu_s r_s / (r_1 + r_s) r_1} - \sqrt{2\mu_s r_s / (r_1 + r_s) r_2}, \\ V_3 &= \sqrt{2\mu_s r_s / (r_1 + r_s) r_2} - \sqrt{2\mu_s r_s / (r_2 + r_s) r_2}. \end{aligned}$$

And this way the expenditures for insertion will be determined by the velocity  $V_s = V_g + |V_2| + |V_3|$ .

In Tables 1 and 2 are given the optimum decisions for the first and second method, respectively, guaranteeing the lowest velocity  $V_s$  with various periods of flight  $\tau$ . The magnitude  $V_g$  was determined for the circular orbit of an Earth

satellite at a height of 200 km. It should be noted that the most optimal solution does not depend on the site of the orbit when it is changed to an infinitely large velocity. In this way the same decisions correspond to the least magnitudes of the sum  $|V_1| + |V_2| + |V_3|$ . Besides this, the least magnitude is also provided by the sum  $|V_2| + |V_3|$ . When  $\tau = 0.5$  years ( $m = 1$ ) the decision is unique. When  $\tau = 20.5$  years  $V_3$  is close to zero. In this case insertion is practically realized with two maneuvers. For all optimum decisions  $k = m$  and  $l = 1$ , i.e., the probe in orbit A completes as many half-turns as the Earth during the entire time of flight  $\tau$ , and only one half-turn in orbit B. In this way orbit A can be regarded as a waiting orbit, in which the probe stays for a period of approximately  $\tau - 0.5$  years from the moment of launching from the orbit of an Earth satellite, and afterward shifts to complete the flight program guaranteeing the execution of the second and third maneuvers.

TABLE 1. CHARACTERISTIC OPTIMUM INTERMEDIATE ORBITS FOR THE FIRST METHOD OF INSERTION, PERIHELION-APHELION

$\tau$ , years	$\tau_a$ , years	$\tau_s$ , years	$r_2$ , AU	$V_1$ , km/sec	$V_2$ , km/sec	$V_3$ , km/sec	$V_4$ , km/sec
0,5	0,245	0,255	0,2599	-10,866	0,254	10,443	18,381
1,5	1,116	0,384	0,6593	-3,375	0,267	3,090	7,090
2,5	2,073	0,426	0,7822	-2,012	0,262	1,744	5,414
3,5	3,052	0,448	0,8421	-1,434	0,258	1,173	4,750
4,5	4,039	0,461	0,8777	-1,114	0,256	0,856	4,395
5,5	5,030	0,470	0,9012	-0,911	0,255	0,655	4,174
10,5	10,011	0,489	0,9541	-0,477	0,252	0,225	3,713
20,5	20,000	0,500	0,9840	-0,244	0,250	-0,006	3,485

Commas indicate decimal points.

## 2. Perturbed Motion

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In its motion on the contra-Earth orbit, the probe gradually goes behind the Sun and becomes invisible to an observer on Earth. For this reason only the perturbed contra-Earth orbits are of practical interest. For the observation of perturbed motion, let us introduce a clockwise system of coordinates  $O\xi\eta\zeta$  (Figure 1), the beginning of which we connect with the unperturbed position of the probe on the contra-Earth orbit. We extend the axis  $O\xi$  from the Sun, and

the axis  $O\xi$  along the kinetic moment of the contra-Earth orbit. Then the plane  $O\eta\xi$  for an observer on Earth will be a flat plane and coordinates  $\eta$  and  $\xi$  will determine the position of the probe on the celestial sphere relative to the center of the solar disc. For unperturbed motion,  $\eta$  and  $\xi$  are identically equal to 0. If we accept the real anomaly  $\vartheta$  as an independent variable, then the motion of the probe in the gravitational field of the Sun in the coordinate system  $O\xi\eta\xi$  will be defined by the following system of equations

$$\begin{aligned} \xi'' + (\ddot{\vartheta}_0/\dot{\vartheta}_0^2)\xi' - \xi - 2\eta' - (\ddot{\vartheta}_0/\dot{\vartheta}_0^2)\eta &= -\frac{r_0}{\rho} \left( \frac{r_0^3}{\Delta^3} (r_0 + \xi) - c \right), \\ \eta'' + (\ddot{\vartheta}_0/\dot{\vartheta}_0^2)\eta' - \eta + 2\xi' + (\ddot{\vartheta}_0/\dot{\vartheta}_0^2)\xi &= -\frac{r_0}{\rho} \frac{r_0^3}{\Delta^3} \eta, \\ \xi'' + (\ddot{\vartheta}_0/\dot{\vartheta}_0^2)\xi' &= -\frac{r_0}{\rho} \frac{r_0^3}{\Delta^3} \xi, \end{aligned} \quad (3)$$

where

$$\Delta = \sqrt{(r_0 + \xi)^2 + \eta^2 + \xi^2},$$

and  $\dot{\vartheta}_0$ ,  $\ddot{\vartheta}_0$  and  $r_0$  are the kinetic characteristics of unperturbed motion:

$$\begin{aligned} (\ddot{\vartheta}_0/\dot{\vartheta}_0^2) &= -2e \sin \vartheta / (1 + e \cos \vartheta), \\ r_0 &= \rho / (1 + e \cos \vartheta), \end{aligned}$$

where  $\rho$  and  $e$  are the parameter and eccentricity of the contra-Earth orbit.

With perturbed motion, the probe should stay in the vicinity of the solar disc, and therefore the coordinates  $\eta$  and  $\xi$ , and consequently  $\xi$  also, should be limited in magnitude by the order of several solar radii. In this case the ratio of the magnitudes of the coordinates to the parameter of the orbit  $\rho$  will be limited by the magnitude 0.1, satisfying the angular declination of the probe on the celestial sphere from the center of the Sun equal to 10 angular radii of the solar disc. Considering this magnitude quite small, we can linearize equation (3) with respect to the ratio between the coordinates and their derivatives as a parameter of orbit  $\rho$ . As the eccentricity of orbit  $e$  has the same order of smallness, we can carry out the linearity relative to the eccentricity. Then we get the following system:

$$\begin{aligned} \xi'' - 3\xi - 2\eta' &= 0, \\ \eta'' + 2\xi' &= 0, \\ \xi'' + \xi &= 0. \end{aligned} \quad (4)$$

As (4) can be transformed into the form

$$\begin{aligned} \ddot{\xi} + \xi' &= 0, \\ \ddot{\eta} + \eta' &= 0, \\ \ddot{\zeta} + \zeta &= 0, \end{aligned} \quad (5)$$

this solution of system (4) can be written in the following way:

$$\xi = \xi_0' \sin \vartheta - (3\xi_0 + 2\eta_0') \cos \vartheta + 4\xi_0 + 2\eta_0', \quad (6)$$

$$\eta = 2\xi_0' \cos \vartheta + (4\eta_0' + 6\xi_0) \sin \vartheta - (3\eta_0' + 6\xi_0) \vartheta - 2\xi_0' \eta_0, \quad (7)$$

$$\zeta = \zeta_0' \sin \vartheta + \zeta_0 \cos \vartheta, \quad (8)$$

where  $\xi_0, \eta_0, \zeta_0, \xi_0', \eta_0', \zeta_0'$  are the initial perturbations of the coordinates and their derivatives where  $\vartheta = 0$ . Equations (7) and (8) determine the parametric trajectory of the perturbed motion of the probe, observed against the celestial sphere from Earth. This trajectory may be considered as an ellipse, the center of which lies on the axis  $O\eta$  and moves with a velocity /10  $(3\eta_0' + 6\xi_0)$ . The magnitudes of the semi-axis of the ellipse and their inclinations toward the axis  $O\eta$  can be arbitrary. In particular the initial perturbations can be chosen in such a way that movement of the center of the ellipse with respect to the Sun can be excluded. For this it is necessary to fulfill the condition

$$\eta_0' = -2\xi_0. \quad (9)$$

When

$$\eta_0 = 2\xi_0' \quad (10)$$

the center of the ellipse will coincide with the center of the solar disc. If we incorporate the condition

$$\begin{aligned} \xi_0 &= \pm 2\xi_0', \\ \zeta_0 &= \pm 2\xi_0', \end{aligned}$$

with conditions (9) and (10), then for an observer on Earth the probe will make a circular motion around the center of the solar disc with a radius equal to  $\sqrt{\xi_0'^2 + \xi_0'^2}$ .

The necessary magnitudes for the initial perturbations can be assured by inserting adjustments into the constituent velocities of the maneuvers inserting the probe into the unperturbed orbit. In this case it is possible to



get arbitrary distinct solutions (6)-(8) with conditions  $\zeta = 0$  where  $\vartheta = 0$ . In order to accept this limitation it is necessary to carry out a supplementary maneuver at an arbitrary, non-apsidal spot on the intermediate orbit.

TABLE 2. CHARACTERISTIC OPTIMUM INTERMEDIATE ORBITS FOR THE SECOND METHOD OF INSERTION, APHELION-PERHELION

$\tau$ , years	$\tau_a$ , years	$\tau_b$ , years	$r$ , AU	$V_1$ , km/sec	$V_2$ , km/sec	$V_3$ , km/sec	$V_4$ , km/sec
0,5	0,255	0,245	0,2599	-10,443	- 0,254	10,866	18,510
1,5	0,757	0,743	1,6207	3,456	- 0,211	- 3,225	7,192
2,5	1,885	0,615	1,3124	2,067	- 0,229	- 1,831	5,475
3,5	2,928	0,572	1,2053	1,476	- 0,236	- 1,236	4,797
4,5	3,949	0,551	1,1508	1,148	- 0,239	- 0,907	4,432
5,5	4,962	0,538	1,1178	0,939	- 0,241	- 0,697	4,204
10,5	9,987	0,513	1,0511	0,492	- 0,246	- 0,246	3,729
20,5	20,000	0,500	1,0173	0,252	- 0,248	- 0,004	3,481

Commas indicate decimal points.

### 3. Insertion Precision

The possible length of use of the probe for investigating circumsolar space is determined by the magnitude of the secular perturbation causing the movement of the center of the elliptic trajectory in respect to the solar disc. With an exact fulfillment of condition (9), secular perturbation will be excluded. If, as a result of errors in carrying out the maneuvers, the initial perturbations  $\xi_0$  and  $\eta_0$  will be different from their computed values by errors of  $\delta\xi_0$  and  $\delta\eta_0$ , while the magnitude of maintaining the center of the elliptical trajectory for a year in harmony with (7) will be equal to

$$\delta\epsilon = -2\pi(3\delta\eta'_0 + 6\delta\xi'_0). \quad (11)$$

The dependence  $\delta\xi_0$  and  $\delta\eta'_0$  on errors in performing maneuvers is determined by the complete program of defining and correcting the insertion trajectory of the probe. If errors in performing the first maneuver are limited to the time the probe stays an intermediate orbit A, and then corrected in carrying out the second and third maneuvers,  $\delta\xi_0$  and  $\delta\eta'_0$  will depend only on errors in carrying out the second and third maneuvers. In this case the linearized dependents  $\delta\xi_0$  and  $\delta\eta'_0$  on mistakes in carrying out these maneuvers can be written in the form [3]:

$$\delta \xi_0 = \frac{2}{\pi} T_b \sqrt{\frac{r_3}{r_2}} \delta V_2, \quad (12)$$

$$\delta \eta_0' = \frac{1}{2\pi} T_g \left( \left( -\frac{r_2}{r_3} - 2 \right) \delta V_2 + \delta V_3 \right), \quad (13)$$

where  $\delta V_2$  and  $\delta V_3$  are errors of magnitude in carrying out the second and third maneuvers respectively, and  $T_b$  and  $T_g$  are periods of the intermediate orbit B and of the contra-Earth orbit respectively. Errors in carrying out these maneuvers in the direction of  $\delta \xi_0$  and  $\delta \eta_0'$  have no effect. Considering  $\delta V_2$  and  $\delta V_3$  independent random magnitudes with a mean square declination  $E_v$  and for the mean square deviation  $E_e$  of annual maintenance  $\delta e$ , we get the following expression

$$E_e = 3 T_g \left( \left( 2 + r_2/r_3 - 2 (T_b/T_g) (r_3/r_2)^{1/2} \right)^2 + 1 \right)^{1/2} E_v.$$

With the given  $E_e$  the magnitude  $E_v$  is practically identical for all trajectories /12 plotted in Tables 1 and 2. For example, if the mean square declination of the annual maintenance  $E_e$  is posited as equal to two radii of the Sun, i.e., one annular radius of the Sun for an earthly observer, then the magnitude  $E_v$  as a function of the chosen length of flight  $\tau$  will be changed within limits from 2.5 to 3 m/sec.

### Conclusions

The insertion of a probe into a contra-Earth orbit is practically feasible with a flight duration equal to 1.5 years. In this situation the total velocity of insertion is approximately the same as the velocity of insertion into the optimum trajectory of flight to Jupiter. An increase in flight length to 5.5 years permits dropping the total velocity of insertion to a magnitude approximately equal to the velocity of sending it to Mars or Venus. A greater increase in flight length will not lead to any significant reduction in the total insertion velocity. The first method of insertion, perihelion-aphelion, requires lower magnitudes of total velocity of insertion than the second method, aphelion-perihelion, with identical flight duration. However, the gain from using the first method is insignificant. A more essential difference in optimum solutions is the fact that for the first method the intermediate orbit A lies within the orbit of the Earth, and for the second method outside the orbit of the Earth. Perturbation of the contra-Earth orbit makes it possible to get an arbitrary elliptical trajectory of probe motion against the celestial sphere,

motionless with reference to the solar disc. The center of this trajectory may coincide with the center of the solar disc. Requirements for exactitude in insertion depend on the necessary time the probe has to remain in the vicinity of the solar disc. When a probe is used for radio sounding for a period of 5 years, precision in carrying out maneuvers must be equal to 0.5 m/sec. With /13 higher precision in carrying out maneuvers and determining the time the probe stays in the vicinity of the solar disc, it is necessary to consider the evolution of the orbits of the Earth and the probe.

## Figures

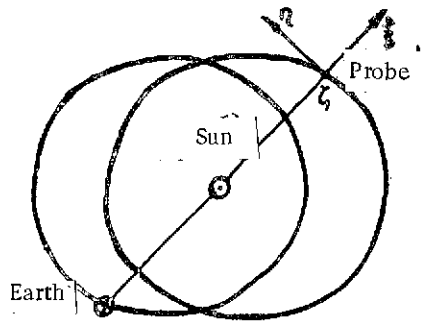


Figure 1. Contra-Earth Orbit and System of Coordinates  $O\xi\eta\zeta$ .

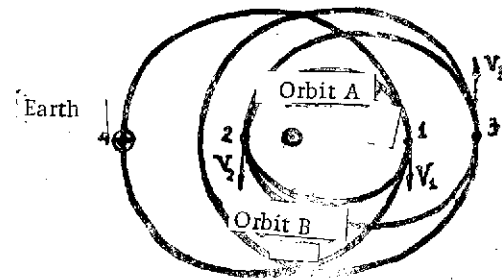


Figure 2. Insertion into Contra-Earth Orbit, First Method.

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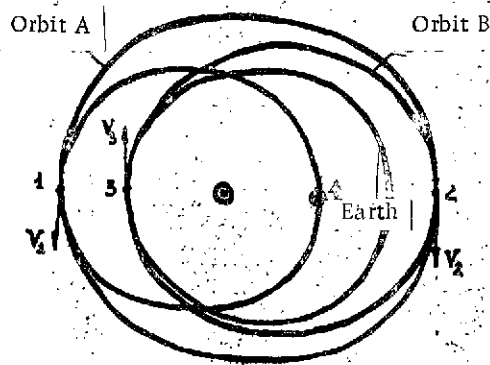


Figure 3. Insertion into Contra-Earth Orbit, Second Method.

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